Department of Mathematics & Philosophy of Engineering

Faculty of Engineering Technology

The Open University of Sri Lanka

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Course: MPZ 3132 – Engineering Mathematics IB

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Instructions

- **Answer all the questions in each assignment.**
- **In these assignments a , b, and c are <u>non – zero, distinct and three digits from the extreme right of your registration number</u>. See the examples**
    - **If the registration number is 30456601 then, since from the extreme right the first digit is 1, therefore $a = 1$. Since the second digit is zero and the third digit is 6 then $b = 6$. Since the fourth digit is also 6 and as a , b, and c are distinct then $c = 5$**
    - **If the registration number is 1036300021 then $a = 1$, $b = 2$ and $c = 3$**
- Write your address on the back page of your answer script.
- Use both sides of the papers to answer the assignments.
- Please send the answer scripts of your assignment on or before the due date to the following address.

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You can collect model answers from the virtual class (www.ou.ac.lk)

User name - student0 Password – MPZ3132

Assignment No.01

Rewrite all the questions substituting your values of a, b and c (Five marks from100)

1. State the Dirichlet's conditions and write down the Fourier series expansion of the period function $f(x)$. The function $f(x)$ is defined such that $f(x) = a + 2bx + 3cx^2$ where $-\pi < x < \pi$ and for the other values $of\ x\ \ f(x) = f(x + 2k\pi)$. Find the Fourier series expansion of $f(x)$. Hence deduce that $\sum_{n=1}^{\infty} \frac{1}{r^2} = \frac{\pi^2}{6}$.

2. The function f is defined such that $f(x) = \begin{cases} xa & , \ 0 \le x \le \frac{b}{2} \\ a(b - x) & , \ \frac{b}{2} \le x \le b \end{cases}$.

   2.1. Extend the above function as an odd periodic function with period $2b$.

   2.2. Draw the extended function in the interval $[-3b, 3b]$.

   2.3. Using half range sine series prove that $f(x) = \frac{4a}{\pi^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)^2} sin\left(\frac{(2n+1)\pi x}{b}\right)$. Hence deduce that $\frac{\pi^2}{8} = \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.

3. The function $f(x)$ is defined such that $f(x) = ax + b$ where $0 < x < 1$.

   3.1. Extend the function $f(x)$

      3.1.1. As an odd periodic function with period 2.

      3.1.2. As an even periodic function with period 2.

      3.1.3. As a periodic function with period 1.

   3.2. Draw the graphs of the above three functions on $[-4, 4]$.

   3.3. Find the Fourier series expansion of $f(x)$

      3.3.1. As a sine series with period 2.

      3.3.2. As a cosine series with period 2.

      3.3.3. As a full trigonometric series with period 1.

4. Write down the Taylor series expansion of $f(x)$ about $x = k$.

   4.1. Find the nth derivative of $f(x) = \frac{1}{a^2x+b^2}$.

   4.2. Find the Taylor series expansion of $f(x) = \frac{1}{a^2x+b^2}$ about $x = 0$.

   4.3. Deduce the Taylor series expansion of $f(x) = \frac{1}{a^2x^2+b^2}$ about $x = 0$.

4.4. By using the integration of the Taylor series find the infinite series for $tan^{-1}\left(\frac{ax}{b}\right)$ about $x = 0$.

4.5. By using the differentiation of the Taylor series find the infinite series for $\frac{2a^2x}{(a^2x^2+b^2)^2}$ about $x = 0$.

5. 5.1. Write down the Taylor polynomial of $f(x)$ about $x = k$.

5.1. The function $f(x)$ is defined such that $f(0) = a$ and $\frac{df(x)}{dx} = bxf(x) + a$. Find the values of $\left(\frac{d^r f(x)}{dx^r}\right)_{x=0}$ for $r = 1, 2, 3, 4$. Hence deduce the order four Taylor polynomial of $f(x)$ about $x = 0$.

5.2. Using the Taylor series expansion find $sin(6a^0)$(If $a = 2$ you have to find the value of $sin(62^0)$) to correct to nine decimal places.

End

Assignment No.02

Rewrite all the questions substituting your values of a, b and c(Five marks from100)

1. Prove that $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$.

1.1. Using the above formula find a particular integral of the differential equation

$$\frac{d^2y}{dx^2} - 2a\frac{dy}{dx} + (a^2 - b^2)y = e^{ax}\cos(cx).$$

1.2. Hence, obtain the general solution of the above differential equation.

2. Using the $D -$ operator methods and the formula $\frac{1}{D+\alpha}f(x) = e^{-\alpha x}\frac{1}{D}e^{\alpha x}f(x)$ solve the system of differential equations $\frac{dx}{dt} + \frac{dy}{dt} + 3x = sint$ and $\frac{dx}{dt} + y - x = cost$.

3. 3.1 The differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = acosx + bsinx$ has a trial function of the form $y_T = Acosx + Bsinx$.

3.1.1. Find the values of A and B.

3.1.2. Hence, obtain the general solution of the above differential equation.

3.2. Using a suitable trail function method find the general solution of the differential equation $\frac{d^2y}{dx^2} + 2a\frac{dy}{dx} + (a^2 + b^2)y = 2012e^{cx}$.

4. 4.1 Find the Laplace transformation of the function $f(t) = \begin{cases} at^2 + bt & , \quad 0 < t < 2 \\ ct & , \quad 2 < t < 3 \\ 7 & , \quad 3 < t \end{cases}$.

4.2. Prove that $L\left(\frac{1}{t}f(t)\right) = \int_s^\infty F(s)ds$. Hence obtain the Laplace transformation of $\frac{e^{at} - cosbt + sinct}{t}$.

4.3. Find the inverse Laplace transformation of $\frac{(a-1)s^2 + (2b+1)as + a(b^2+c^2)}{(s-a)(s^2+2bs+b^2+c^2)}$.

5. Using the Laplace transformation solve the following differential equations

5.1. $\frac{dx}{dt} + a^2y = 0 \quad and \ \frac{dy}{dt} - b^2x = 0$. Where $x(0) = y(0) = 1$.

5.2. $\frac{d^2y}{dx^2} + b^2y = sinax$ where $y(0) = y'(0) = 0$.

End

Assignment No.03

Rewrite all the questions substituting your values of a, b and c(Five marks from100)

1. 1.1 Shade the regions satisfying the following inequalities

$|z - a(1 + i)| \leq a \ and - a \leq Im(z) - Re(z) \leq a$.

1.2. Find $Log(a + ib)$ of the form $x + iy$ where $x , y \in R$

1.3. The function f is defined as follows $f(z) = \frac{z^2}{a+ib}$ where $|z| \leq c \ and \ 0 < \arg(z) \leq \frac{\pi}{2}$

1.3.1. Determine the image of the above function.

1.3.2. Draw the domain and image of the above function.

1.4. The function g is defined as $g(z) = \frac{az+b}{cz+1}$. Show that $1 - 1$ and find the inverse of g.

2. 2.1 Define the reduced row echelon form of a matrix.

2.2. Consider the matrix $A = \begin{bmatrix} 3 & -2 & 1 \\ -11 & 8 & 1 \\ 10 & -7 & 1 \\ -1 & 1 & 2 \end{bmatrix}$

2.2.1. Applying elementary row operations find $RREM(A)$.

2.2.2. Considering $RREM(A)$ solve the following system of linear equations.

$$3x - 2y + z = 0$$

$$-11x + 8y + z = 0$$

$$10x - 7y + z = 0$$

$$-x + y + 2z = 0$$

2.3. Using the elementary operations find the rank of $\begin{bmatrix} 1 & a & b \\ 2 & 2a + b - 1 & a + 2b \\ 5 & 5a + 3b - 3 & 3a + 5b \end{bmatrix}$.

3. By using the augmented matrix and elementary operations, explaining the nature of the solutions, find the solutions of the following system of linear equations

$$x + 2y - 3z = 4$$

$$3x - y + 5z = 2$$

$$4x + y + (k^2 - 14)z = 2 - k$$

4. 4.1 A particle moves along the curve $x = at^4 + 2,\ y = bt^2\ ,\ z = ct + 8$ where t is the time. Find the components of its velocity and acceleration of the particle at the time $t = 1$ in the direction of $2\boldsymbol{i} - 3\boldsymbol{j} + 6\boldsymbol{k}.$

4.2. If $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{a}$ are the position vector, velocity and acceleration of a particle at time t respectively. Prove that

$$\frac{d}{dt}\big(a\boldsymbol{v} \times (b\boldsymbol{r} \times c\boldsymbol{v})\big) = abc[2(\boldsymbol{v} \cdot \boldsymbol{a})\boldsymbol{r} - (\boldsymbol{r} \cdot \boldsymbol{a})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{r})\boldsymbol{a}].$$

4.3. Prove that if $\boldsymbol{f}(t) = \frac{e^{at} sinbt}{\sqrt{1+e^{2at}}}\mathbf{i} + \frac{e^{at} cosbt}{\sqrt{1+e^{2at}}}\mathbf{j} + \frac{1}{\sqrt{1+e^{2at}}}\mathbf{k}$

4.3.1. $|\boldsymbol{f}(t)| = 1.$

4.3.2. $\boldsymbol{f}(t)$ is perpendicular to $\frac{d}{dt}(\boldsymbol{f}(t))$.

4.3.3. $\boldsymbol{f}(t) \cdot \frac{d^2(\boldsymbol{f}(t))}{dt^2} = -\left|\frac{d(\boldsymbol{f}(t))}{dt}\right|^2.$

5. Prove that the moment generating function of the normal distribution which has the probability density function $f(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ where $x \in R$ is $M(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$.

   Hence find the mean and variance of the normal distribution.

   Define the binomial distribution. If $X \sim Bin(n\,, p)$ prove that the moment generating function of X is $(pe^t + 1 - p)^n$ Hence prove that the mean and variance of X are np and $np(1-p)$ respectively.

   5.1. Prove the formulae for the expectation and variance of the above distribution.

   5.2. Explain the normal approximation for the above distribution.

   5.3. It is known that in a sack of mixed paddy seeds 35% are red. Use the normal approximation to the binomial distribution to find the probability that in a sample of 400 seeds there are

      5.3.1. Less than 120 red seeds.

      5.3.2. Between 120 and 150 (including 120 ,150) red seeds.

      5.3.3. More than 160 red seeds.

End

Assignment No.04

Rewrite all the questions substituting your values of a, b and c(Five marks from100)

1. 1.1 Find the moments of inertia of the following bodies about the given axis.

      1.1.1. A uniform rod AB of mass m and length $2l$, the axis perpendicular to the rod passing through the midpoint of the rod.

      1.1.2. A uniform circular ring mass m and radius l, the axis perpendicular to the plane of the ring passing through the center.

      1.1.3. A uniform circular disc of radius l, the axis perpendicular to the plane of the disc passing through the center.

      1.1.4. A uniform solid sphere of radius l , the axis through the center of the sphere

   1.2. Using a suitable theorem,

      1.2.1. Deduce the moments of inertia of the above rod about the axis perpendicular to the rod passing through A.

1.2.2. Deduce the moments of inertia of the disc of radius l mass aM about the axis perpendicular to the plane of disc passing through the point of $3l$ distance of the center of the disc.

1.2.3. Deduce the moment of inertia of a uniform sphere about the tangent to the sphere.

2. A pendulum consists of a uniform rod AB, of mass am and length $2l$ fixed at B to a point on the circumference of a uniform circular disc of mass bM and radius l. The disc and the line of the rod lie in the same vertical plane with the center of the disc lying on the rod produced. The pendulum is free to rotate in a vertical plane about a fixed horizontal axis through A perpendicular to the plane of the disc.

   2.1. Taking the results of the question no.1 find the moments of inertia of the pendulum about the axis perpendicular to the plane of the disc passing through A.

   2.2. When the rod is horizontal the disc is projected with an angular velocity ω in the downward vertical direction. Find the angular velocity after the rod turns an angle θ. If the system rotates complete circles prove that

$$\omega^2 \geq \frac{12g(am + 3bM)}{l(8am + 57bM)}$$

   2.3. Find the period of small oscillations of the pendulum.

3. A particle is projected vertically upwards with speed u from a point O. The air resistance on a unit mass is $g\left(\frac{b^2v^4}{a^2u^4}\right)$ where v is the velocity of the particle at a distance x from O. Prove that $\frac{d}{dx}(v^2) = -2g\left(\frac{a^2u^4+b^2v^4}{b^2u^4}\right)$. Deduce that the maximum height attained by the particle is $\frac{u^2}{2g}tan^{-1}\left(\frac{b}{a}\right)$. Find the speed of the particle when it returns to O.

4. A particle P moves on a plane. At time t the polar coordinates of P is (r, θ) and the position vector of P is $\boldsymbol{r}$. If the unit vector along $\boldsymbol{r}$ is $\boldsymbol{l}$ and the unit vector perpendicular to $\boldsymbol{r}$ and θ increasing direction is $\boldsymbol{m}$.

   4.1. Prove that the velocity $\boldsymbol{v} = \dot{r}\boldsymbol{l} + r\dot{\theta}\boldsymbol{m}$ & the acceleration

$$\boldsymbol{f} = (\ddot{r} - r\ddot{\theta})\boldsymbol{l} + \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})\boldsymbol{m}.$$

4.2. Particles P and Q of masses am and bM respectively are connected by an inextensible weightless string of length $2l$. The string passes through a smooth small fixed ring O on a smooth horizontal plane. At initially the string is taut and $OP = l$. Then P is projected on the plane with speed cu perpendicular to OP. After time $t, = r\ (< 2l)$. Prove that the tension of the string is $\frac{aml^2c^2u^2}{(am+bM)r^3}$.

5. A uniform rectangular lamina $ABCD$ is immersed vertically in a homogeneous liquid such that AB and CD are horizontal. The distances to AB and CD from the surface of the liquid are h_1 and h_2 respectively. Prove that that the centre of pressure of the lamina is at the distance $\frac{2}{3}\left(\frac{h_1^2+h_1h_2+h_2^2}{h_1+h_2}\right)$ from the free liquid surface.

5.1. The length of a side the square base and height of a thin pyramid are $6a$ and $4a$ respectively. The pyramid is completely filled with a homogeneous liquid with weight w. The weight of the square base is W. The pyramid is kept such that a triangular surface touching a horizontal floor.

5.1.1. Find the resultant reaction on the triangular surfaces of the pyramid.

5.1.2. If the square is a door such that the highest edge of the square is smoothly hinged and the midpoint of the lowest edge is the lock of the door find the reaction on the lock.

End